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Magnetized Domain Walls Cosmological Model in General Relativity

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Abstract A non-static Bianchi type-III domain walls cosmological models in presence and absence of magnetic field are investigated in general theory of relativity. We assume that F_{12} is only the non-vanishing component of F_{ij}. To obtain deterministic model, we assume relations $B = C^n$ and $\rho = p$. Some physical properties of these models are discussed.

Keywords Bianchi type-III space-time · Magnetic field · Domain walls

1 Introduction

Bianchi type-III cosmological models are the simplest anisotropic universe models playing an important role in understanding essential features of the universe. In this class of models it is possible to accommodate the topological defects (cosmic strings, domain walls, monopoles etc.). At the very early stage of evolution of universe, it is generally assumed that during the phase transition the symmetry of the universe is broken spontaneously. It can give rise to topologically stable defects such as strings, domain walls and monopoles [10]. In particular, the domain walls have become important in recent years from cosmological stand point when a new scenario of galaxy formation has been proposed by Hill et al. [7]. According to them the formation of galaxies are due to domain walls produced during phase transitions after the time of recombination of matter and radiation. In the case in which the phase transition is induced by Higgs sector of the standard model, the defects are domain walls across which the field flips from one minimum to other. The defect density is then related to the domain size and the dynamics of the domain walls is governed by the surface tension. It is clear that a full analysis of the role of domain walls in the universe imposes the study of their interaction with particles in the primordial plasma.

So far a considerable amount of work has been done on domain walls. Vilenkin [18], Isper and Sikivie [8], Windraw [19], Goetz [5], Mukherjee [14], Rahmann [15], Reddy and

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Subba Rao [16], Adhav et al. [1] are some of the authors who have investigated several aspects of domain walls.

Also the occurrence of magnetic fields on galactic scale is well established fact today, and their importance for a variety of astrophysical phenomena is generally acknowledged as pointed out by Zel'dovich et al. [20]. Harrison [6] has suggested that magnetic field could have a cosmological origin. As a natural consequence, we should include magnetic field in the energy-momentum tensor of the universe. The presence of primoradial magnetic fields in the early stage of the evolution of the universe has been discussed by Misner et al. [13], Asseo and Sol [2], Kim et al. [11], Melvin [12]. Also Iwazaki [9] and Cea and Tedesco [4] gives the interesting phenomena as the magnetization of domain walls and the dynamical generation of massive ferromagnetic domain walls.

The object of this paper is to investigate the Bianchi type-III domain walls cosmological model in the presence and absence of a magnetic field in general relativity. To get a realistic model, we have assumed a condition $B = C^n$ between the metric potentials B and C which are the functions of time alone and $p = \rho$.

2 The Metric and the Field Equations

The spatially homogeneous and anisotropic plane symmetric Bianchi type-III line element is given by

$$ds^2 = dt^2 - A^2 dx^2 - B^2 e^{-2ax} dy^2 - C^2 dz^2, \tag{2.1}$$

where a is non zero constant. A, B, C are functions of time 't'.

A thick domain wall can be viewed as a soliton like solution of the scalar field equations coupled with gravity. There are two ways of studying thick domain walls. One way is to solve gravitational field equations with an energy momentum tensor describing a scalar field φ with self-interactions contained in potentials $V(\psi)$ given by

$$\psi,_i \psi,_j - g_{ij}\left[\frac{1}{2}\psi,_k \psi^{,k} - V(\psi)\right].$$

Second approach is to assume the energy momentum tensor in of the form

$$T_{ij} = \rho(g_{ij} + \omega_i \omega_j) + p\omega_i \omega_j, \quad \omega^i \omega_j = -1.$$

The energy momentum tensor for a system of domain walls and magnetic field is given by

$$T_{ij} = \rho(g_{ij} + \omega_i \omega_j) + p\omega_i \omega + E_{ij}, \quad \omega^i \omega_j = -1, \tag{2.2}$$

where ρ is the energy density of the wall, p is the pressure in the direction normal to the plane of the wall and ω_i is a unit space-like vectors in the same direction.

Here we use the second approach to study the thick domain walls.

The electromagnetic field E_{ij} is given by

$$E_{ij} = \frac{1}{4\pi}\left[-F_{is} F_{jp} g^{sp} + \frac{1}{4} g_{ij} F_{sp} F^{sp}\right]. \tag{2.3}$$

We assume that F_{12} is the only non-vanishing component of F_{ij}.

The Maxwell's equation

$$\frac{\partial}{\partial x^j}\left(F_{ij}\sqrt{-g}\right)=0$$

leads to

$$\frac{\partial}{\partial y}\left(F^{12}ABCe^{-ax}\right)=0$$
$$\Rightarrow \quad F_{12}=He^{-ax}, \quad \text{where } H \text{ and } a \text{ are constants.} \tag{2.4}$$

We consider F_{12} as the only non-vanishing component of F_{ij} because cosmological model which contains a global magnetic field is necessarily anisotropic since the magnetic field vector specifies a preferred spatial direction [3]. We assume that the current is flowing along z-axis, so magnetic field is in the xy-plane. Thus F_{12} is the only non-vanishing component of F_{ij}. In the same way, other components F_{13}, F_{23} of F_{ij} may be assumed non-zero but it will create more complexity. Thus when $F_{12}\neq 0$ then $F_{13}=F_{23}=0$ and $F_{14}=F_{24}=F_{34}=0$ due to the assumption of infinite electrical conductivity [17]. If finite conductivity is assumed then $F_{14}\neq 0$, $F_{24}\neq 0$, $F_{34}\neq 0$. In these situations, the problem become more complex to solve Einstein's field equation. The reference in the text involve such type of assumptions to avoid the complexity of the problem. To get realistic model of the universe, we take F_{12} different from zero.

The non-vanishing components of E_{ij} corresponding to the line element (2.1) are as follows:

$$E_1^1=\frac{-H^2}{8\pi A^2B^2}=E_2^2=-E_3^3=-E_4^4 \quad \text{and} \quad E_1^4=0. \tag{2.5}$$

The Einstein field equations are

$$R_i^j-\frac{1}{2}Rg_i^j=-8\pi T_i^j. \tag{2.6}$$

The field equations (2.6) for the line element (2.1) with (2.2), (2.3), (2.4) and (2.5) will be

$$\frac{B_{44}}{B}+\frac{C_{44}}{C}+\frac{B_4}{B}\frac{C_4}{C}=8\pi\rho-\frac{H^2}{A^2B^2}, \tag{2.7}$$

$$\frac{A_{44}}{A}+\frac{C_{44}}{C}+\frac{A_4}{A}\frac{C_4}{C}=8\pi\rho-\frac{H^2}{A^2B^2}, \tag{2.8}$$

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{A_4}{A}\frac{B_4}{B}-\frac{a^2}{A^2}=-8\pi p+\frac{H^2}{A^2B^2}, \tag{2.9}$$

$$\frac{A_4B_4}{AB}+\frac{A_4C_4}{AC}+\frac{B_4}{B}\frac{C_4}{C}-\frac{a^2}{A^2}=8\pi\rho+\frac{H^2}{A^2B^2}, \tag{2.10}$$

$$\frac{A_4}{A}-\frac{B_4}{B}=0, \tag{2.11}$$

where the subscript '4' after A, B and C denote ordinary differentiation with respect to t.

From (2.11), we have

$$A=\mu B, \quad \text{where } \mu \text{ is a constant of integration.} \tag{2.12}$$

From (2.9), (2.10) and (2.12), we have

$$\frac{B_{44}}{B} + \left(\frac{B_4}{B}\right)^2 + \frac{B_4 C_4}{BC} - \frac{a^2}{\mu^2 B^2} = 4\pi(-p+\rho) + \frac{H^2}{\mu^2 B^4}. \tag{2.13}$$

Equations (2.7) to (2.10) are four independent equations in six unknowns A, B, C, ρ, p, H. Hence to find a determinate solution we use two conditions:

$$\text{(i)} \quad \rho = p \quad \text{and} \quad \text{(ii)} \quad B = C^n. \tag{2.14}$$

From (2.13) and (2.14), we have

$$\frac{C_{44}}{C} + 2n\left(\frac{C_4}{C}\right)^2 = \frac{H^2}{n\mu^2 C^{4n}} + \frac{a^2}{n\mu^2 C^{2n}},$$

which again leads to

$$CC_{44} + 2nC_4^2 = \frac{H^2}{n\mu^2 C^{4n-2}} + \frac{a^2}{n\mu^2 C^{2n-2}}. \tag{2.15}$$

Let us consider

$$\begin{aligned} C_4 &= f(C), \\ C_{44} &= ff' \quad \text{where } f' = \frac{df}{dC}. \end{aligned} \tag{2.16}$$

With the help of (2.16), (2.15) reduces to

$$2ff' + 4n\frac{f^2}{C} = \frac{2H^2}{n\mu^2 C^{4n-1}} + \frac{2a^2}{n\mu^2 C^{2n-1}}. \tag{2.17}$$

On simplifying (2.17), we have

$$f^2 = \frac{H^2}{n\mu^2}C^{2-4n} + \frac{a^2}{n\mu^2}\frac{C^{2-2n}}{(n+1)} + k_1 C^{-4n} \tag{2.18}$$

where k_1 is constant of integration. But

$$f = C_4. \tag{2.19}$$

Using (2.19), (2.18) becomes

$$\frac{C^{2n-1}}{\sqrt{\beta^2 + \frac{a^2}{(n^2+n)\mu^2}C^{2n} + k_1 C^{-2}}} dC = dt, \tag{2.20}$$

where

$$\beta^2 = \frac{H^2}{n\mu^2}. \tag{2.21}$$

To get determinate solution, we assume $k_1 = 0$.

Therefore (2.20) reduces to

$$\beta^2 + \frac{a^2 C^{2n}}{(n^2+n)\mu^2} = \frac{(t+k_2)^2 a^4}{(n+1)^2\mu^4},$$

$$C^{2n} = (t+k_2)^2 \frac{a^2(n^2+n)}{\mu^2(n^2+1)^2} - \beta^2 \frac{(n^2+n)}{a^2}\mu^2. \tag{2.22}$$

From (2.12), (2.14) and (2.22), we have

$$A = \mu\left[k_3^2(t+k_2)^2 - \frac{\beta^2}{k_3^2}k_4\right]^{\frac{1}{2}},$$

$$B = \left[k_3^2(t+k_2)^2 - \frac{\beta^2}{k_3^2}k_4\right]^{\frac{1}{2}}, \tag{2.23}$$

$$C = \left[k_3^2(t+k_2)^2 - \frac{\beta^2}{k_3^2}k_4\right]^{\frac{1}{2n}},$$

where

$$k_3^2 = \frac{n^2+n}{(n^2+1)^2}\frac{a^2}{\mu^2} \quad \text{and} \quad k_4 = \frac{(n^2+n)^2}{(n^2+1)^2}.$$

Using (2.23), the line element (2.1) becomes

$$ds^2 = dt^2 - \mu^2\left[k_3^2(t+k_2)^2 - \frac{\beta^2}{k_3^2}k_4\right]dx^2 - \left[k_3^2(t+k_2)^2 - \frac{\beta^2}{k_3^2}k_4\right]e^{-2ax}dy^2$$
$$- \left[k_3^2(t+k_2)^2 - \frac{\beta^2}{k_3^2}k_4\right]^{\frac{1}{n}} dz^2.$$

After a proper choice of coordinates and constants ,the above equation can be written as

$$ds^2 = dT^2 - \mu^2\left[k_3^2(T)^2 - \frac{\beta^2}{k_3^2}k_4\right]dX^2 - \left[k_3^2(T)^2 - \frac{\beta^2}{k_3^2}k_4\right]e^{-2ax}dY^2$$
$$- \left[k_3^2(T)^2 - \frac{\beta^2}{k_3^2}k_4\right]^{\frac{1}{n}} dZ^2. \tag{2.24}$$

Physical Properties The important physical quantities in cosmology, proper volume v, expansion scalar θ, shear scalar σ^2 for the model (2.24) are given by

$$v = \sqrt{-g} = \mu^2\left[k_3^2 T^2 - \frac{\beta^2}{k_3^2}k_4^2\right]^{\frac{2n+1}{2n}} e^{-ax},$$

$$\theta = \frac{2n+1}{3n}\left[\frac{k_3^2 T}{k_3^2 T^2 - \frac{\beta^2}{k_3^2}k_4^2}\right]^2,$$

$$\sigma^2 = \frac{1}{6}\left(\frac{2n+1}{3n}\right)^2 \left[\frac{k_3^2 T}{k_3^2 T^2 - \frac{\beta^2}{k_3^2} k_4^2}\right]^2,$$

$$8\pi p = \left(\frac{4n+1}{4n}\right) \frac{k_3^4}{[k_3^2 (T)^2 - \frac{\beta^2}{k_3^2} k_4]^2}.$$

There is a big-bang in the model at $T = \frac{\beta}{k_3^2} k_4$. The expansion in the model decreases as time increases. The model in general represents shearing and non-rotating model.

Since $\lim_{T\to\infty}(\frac{\sigma}{\theta}) \neq 0$, the model does not approach isotropy for large value of T.

If $n = -\frac{1}{2}$, then the expansion in the model stops.

When $T \to \frac{\beta}{k_3^2} k_4$ then pressure p and density ρ tends to infinite value.

3 In Absence of Magnetic Field

In the absence of magnetic field $\beta \to 0$, the corresponding metric reduces to the following form:

$$ds^2 = dt^2 - \mu^2 k_3^2 (t + k_2)^2 dx^2 - k_3^2 (t + k_2)^2 e^{-2ax} dy^2 - k_3^2 (t + k_2)^2 dz^2. \quad (3.1)$$

By using the transformations

$$k_3 \mu x = X, \quad k_3 y = Y, \quad k_3^{\frac{1}{n}} z = Z, \quad t + k_2 = T,$$

the metric (3.1) reduces to the form

$$ds^2 = dT^2 - T^2 dX^2 - T^2 e^{-\frac{2ax}{k_3 \mu}} dY^2 - T^{\frac{2}{n}} dZ^2. \quad (3.2)$$

Physical Properties The proper volume, scalar expansion and shear scalar for the model (3.2) are given by

$$v = \sqrt{-g} = T^{\frac{2n+1}{2n}} e^{\frac{-ax}{k3\mu}},$$

$$\theta = \left(\frac{2n+1}{3n}\right) \frac{1}{T},$$

$$\sigma^2 = \frac{1}{6}\left(\frac{2n+1}{3n}\right)^2 \frac{1}{T^2},$$

$$8\pi p = \left(\frac{4n+1}{4n}\right) \frac{1}{T^4}.$$

It may be observed that at initial moment (when $T = 0$) the proper volume will be zero. When $T \to 0$, the expansion scalar θ, shear scalar σ^2 tends to infinity.

For large values of T ($T \to \infty$) we observe that proper volume will be infinite while expansion scalar θ and shear scalar σ^2 becomes zero.

Also $\lim_{T\to\infty}(\frac{\sigma}{\theta}) \neq 0$, the model does not approach isotropy.

4 Conclusion

In this paper, we have obtained non-static Bianchi type-III cosmological domain wall model in presence and absence of magnetic field. The model is expanding, shearing and non-vanishing. Thick domain walls with magnetic field and space times associated with them have cosmological interest due to their important applications in structure formation of the universe.

References

1. Adhav, K.S., Nimkar, A.S., Naidu, R.L.: Astrophys. Space Sci. **312**, 165–169 (2007)
2. Asseo, E., Sol, H.: Phys. Rev. **6**, 148 (1987)
3. Bronnikov, K.A., Chudayeva, E.N., Shikin, G.N.: Class. Quantum Gravity **21**, 3389 (2004)
4. Cea, P., Tedesco, L.: Phys. Lett. B **450**, 61 (1999)
5. Goetz, G.: J. Math. Phys. **31**, 2683 (1990)
6. Harisson, E.R.: Phys. Rev. Lett. **30**, 188 (1973)
7. Hill, C.T., Schramm, D.N., Fry, J.N.: Comments Nucl. Part. Phys. **19**, 25 (1989)
8. Isper, J., Sikivie, P.: Phys. Rev. D. **30**, 712 (1984)
9. Iwazaki, I.: Phys. Rev. D **56**, 2435 (1997)
10. Kibble, T.W.: J. Phys. A. Math. Gen. **9**, 1387 (1976)
11. Kim, K.T., Tribble, P.G., Kronberg, P.P.: Astrophys. J. **379**, 80 (1991)
12. Melvin, M.A.: Ann. NY Acad. Sci. **262**, 253 (1975)
13. Misner, C.W., Thorne, K.S., Wheeler, J.A.: Gravitation. Freeman, New York (1973)
14. Mukherjee, M.: Class. Quantum Gravity **10**, 131 (1993)
15. Rahmann, F.: Astrophys. Space Sci. **282**, 625 (2002)
16. Reddy, D.R.K., Subba Rao, M.V.: Astrophys. Space Sci. **302**, 157–160 (2006)
17. Roy, M.: Pramana (J. Phys. Indian Acad. Sci.) **55**(4), 576 (2000)
18. Vilenkin, A.: Phys. Rev. D. **23**, 852 (1981)
19. Windraw, L.M.: Phys. Rev. D. **39**, 3571 (1989)
20. Zel'dovich, Y.B., Ruzmainkin, A.A., Sokoloff, D.D.: Magnetic Field in Astrophysics. Jordan and Breach, New York (1993)